

AN OBSERVED FUNDAMENTAL PLANE RELATION FOR SUPERMASSIVE BLACK HOLES

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ABSTRACT

We study observed correlations between supermassive black hole (BHs) and the properties of their host galaxies, and show that the observations define a BH “fundamental plane” (BHFP), of the form $M_{\text{BH}} \propto \sigma^{3.0 \pm 0.3} R_e^{0.43 \pm 0.19}$ or $M_{\text{BH}} \propto M_*^{0.54 \pm 0.17} \sigma^{2.2 \pm 0.5}$, analogous to the FP of elliptical galaxies. The BHFP is preferred over a simple relation between M_{BH} and any of σ , M_* , M_{dyn} , or R_e alone at $> 3\sigma$ (99.9%) significance. The existence of this BHFP has important implications for the formation of supermassive BHs and the masses of the very largest black holes, and immediately resolves several apparent conflicts between the BH masses expected and measured for outliers in both the $M_{\text{BH}} - \sigma$ and $M_{\text{BH}} - M_*$ relations.

Subject headings: quasars: general — galaxies: active — galaxies: evolution — cosmology: theory

1. INTRODUCTION

Discoveries of correlations between the masses of supermassive black holes (BHs) in the centers of nearby galaxies and the properties of their host spheroids (e.g., Kormendy & Richstone 1995) demonstrate a fundamental link between the growth of BHs and galaxy formation. A large number of similar correlations have now been identified, linking BH mass to host luminosity (Kormendy & Richstone 1995), mass (Magorrian et al. 1998), velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000), concentration or Sersic index (Graham et al. 2001; Graham & Driver 2006), and binding energy (Aller & Richstone 2007), among others. However, because these properties of host spheroids are themselves correlated, it is not clear whether any are in some sense more basic (see e.g. Novak et al. 2006, for such a comparison).

The lack of a clear motivation for favoring one relation over another has led to substantial observational and theoretical debate over the “proper” correlation for systems which may not lie on the mean correlation between host properties, and over the demographics of the most massive BHs (e.g., Bernardi et al. 2006; Lauer et al. 2006b; Batcheldor et al. 2006; Wyithe 2006). One possibility is that these different correlations are projections of the same “fundamental plane” (FP) relating BH mass with two or more spheroid properties such as stellar mass, velocity dispersion, or effective radius, in analogy to the well-established fundamental plane of spheroids. For the case of spheroids, it is now understood that various correlations, including the Faber-Jackson relation (Faber & Jackson 1976) between luminosity (or effectively stellar mass M_*) and velocity dispersion σ , the Kormendy (1977) relation between effective radius R_e and surface brightness I_e , and the size-luminosity or size-mass relations (e.g., Shen et al. 2003) between R_e and M_* , are all projections of a fundamental plane relating $R_e \propto \sigma^\alpha I_e^\beta$ (Dressler et al. 1987; Djorgovski & Davis 1987).

In their analysis of the relation between BH mass and host luminosity or dynamical mass, M_{dyn} , Marconi & Hunt (2003)

(see also de Francesco et al. 2006) noted that the residuals of the $M_{\text{BH}} - \sigma$ relation (effectively M_{BH}/σ^4 ; Tremaine et al. 2002) were significantly correlated with the effective radii of the systems in their sample. Figure 1 shows this, for the compilation of observations described in § 2 – there is indeed a clear trend that systems with larger effective radii or stellar masses tend to have larger $M_{\text{BH}}/\langle M_{\text{BH}}(\sigma) \rangle$. In that context, the authors argued for this as evidence favoring a relation between M_{BH} and $M_{\text{dyn}} \propto \sigma^2 R_e$ over $M_{\text{BH}} \propto \sigma^4$, but it is not clear that a dependence on M_{dyn} alone completely or accurately captures the behavior in these residuals (and in § 3 we show that it does not). Furthermore, finding $M_{\text{BH}}/\sigma^4 \propto R_e^\beta$ does not necessarily imply a FP-like relation, if the correlation between M_{BH} and σ or σ and R_e has some nonlinear (e.g., Wyithe 2006) or otherwise incompletely accounted-for behavior. Still, this brings up the important possibility of a true FP-like relation in which the combination of two properties such as M_* and σ determines M_{BH} , which we study herein.

Throughout, we adopt a $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ cosmology (and correct all observations accordingly), but note this choice has little effect on our conclusions.

2. THE DATA

We consider the sample of local BHs for which masses have been reliably determined via either kinematic or maser measurements. Specifically, we adopt the sample of 38 local systems for which values of M_{BH} , σ , R_e , M_{dyn} , and bulge luminosities are compiled in Marconi & Hunt (2003) and Häring & Rix (2004) (see also Magorrian et al. 1998; Merritt & Ferrarese 2001; Tremaine et al. 2002). We adopt the dynamical masses from the more detailed Jeans modeling in Häring & Rix (2004). We estimate the total stellar mass M_* from the total K -band luminosity given in Marconi & Hunt (2003), using the K -band mass-to-light ratios as a function of luminosity from Bell et al. (2003) (specifically assuming a “diet” Salpeter IMF, although this only affects the absolute normalization of the relevant relations). We have repeated these calculations using mass-to-light ratios estimated independently for each object from their $B - J$ colors or full $UBVJHK$ photometry, and find it makes little difference. Where possible, we update measurements of R_e and σ with more recent values from Lauer et al. (2005, 2006a); McDermid et al. (2006) and from Kormendy et al. (2007).

Although it should only affect the normalization of the re-

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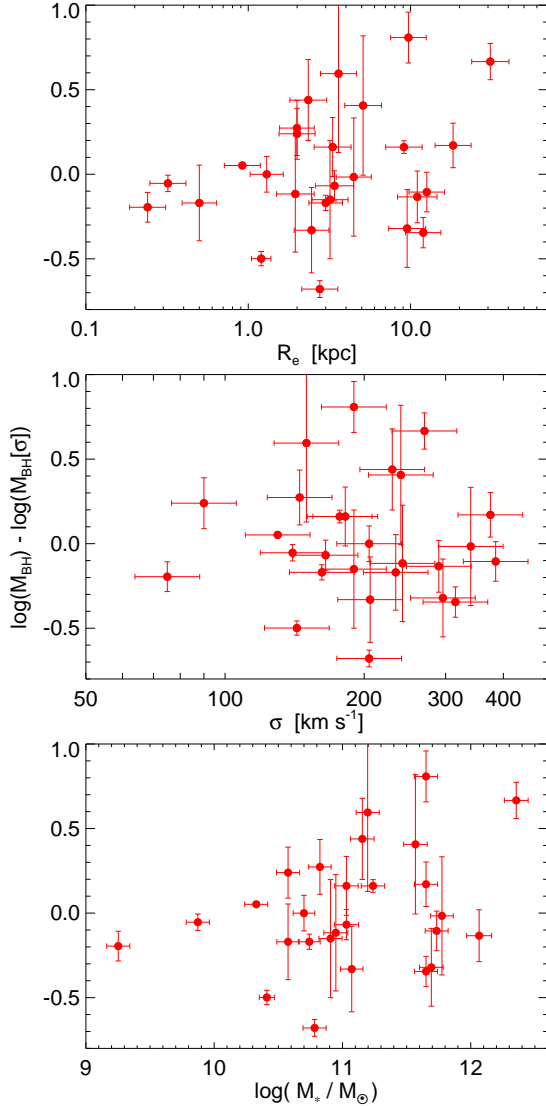


FIG. 1.— Observed residual in BH mass relative to the best-fit $M_{\text{BH}}-\sigma$ relation, as a function of the host galaxy effective radius R_e , velocity dispersion σ , and stellar mass M_* . There is a clear suggestion in the data of a residual correlation with R_e and M_* , which we examine in detail in Figure 2.

lations herein, we note that our adopted cosmology is identical to that used to determine all quoted values in these works. When we fit the observations to e.g. the mean $M_{\text{BH}}-\sigma$ relation and other BH-host relations, we consider only the subsample of 27 objects in Marconi & Hunt (2003) which are deemed to have ‘secure’ BH and bulge measurements (i.e. for which the BH sphere of influence is clearly resolved, the bulge profile can be well-measured, and maser spots (where used to measure M_{BH}) are in Keplerian orbits). Our results are not qualitatively changed if we consider the entire sample in these fits, but their statistical significance is somewhat reduced.

3. A BLACK HOLE FUNDAMENTAL PLANE

We wish to determine whether or not a simple one-to-one correlation between e.g. M_{BH} and σ is a sufficient description of the data, or if there is evidence for additional dependence on a second parameter such as R_e or M_* . The most efficient way to determine such a dependence is by looking for correlations between the residuals of the various projections of such a potential BHFP relation. In Figure 1, we follow Marconi & Hunt (2003) and plot the dependence on the

residual of the $M_{\text{BH}}-\sigma$ relation on R_e and M_* . This provides a clear suggestion of a residual dependence, and motivates us to examine a more complete description than a simple $M_{\text{BH}}-\sigma$ relation. However, in order to robustly do so, we must consider the correlations between residuals at fixed σ , not the correlation between e.g. the residual of $M_{\text{BH}}-\sigma$ and the actual value of a quantity such as R_e or M_* .

Figure 2 plots the correlation between BH mass M_{BH} and host bulge effective radius R_e or bulge stellar mass M_* , all at fixed σ . Specifically, we determine the residual with respect to the $M_{\text{BH}}-\sigma$ relation by fitting $M_{\text{BH}}(\sigma)$ to an arbitrary log-polynomial

$$\langle \log(M_{\text{BH}}) \rangle = \sum [a_n \log(\sigma)^n], \quad (1)$$

allowing as many terms as the data favor (i.e. until $\Delta\chi^2$ with respect to the fitted relation is < 1), and then taking

$$\Delta \log(M_{\text{BH}} | \sigma) \equiv \log(M_{\text{BH}}) - \langle \log(M_{\text{BH}}) \rangle(\sigma). \quad (2)$$

We determine the residual $\Delta \log(R_e | \sigma)$ (or $\Delta \log(M_* | \sigma)$, for the stellar mass) in identical fashion, and plot the correlation between the two. We allow arbitrarily high terms in $\log(\sigma)$ to avoid introducing bias by assuming e.g. a simple power-law correlation between M_{BH} and σ , but find in practice that such terms are not needed – as discussed below, there is no significant evidence for a log-quadratic (or higher-order) dependence of M_{BH} on σ , R_e , or M_* , so allowing for these terms changes the residual best-fit solutions in Figure 2 by $\ll 1\sigma$. For reference, we find (in this sample) a best-fit correlation between $R_e \propto \sigma^{2.0 \pm 0.4}$ (ignoring the log-quadratic and higher order terms, which are not significant), similar to that found for all SDSS galaxies by Shen et al. (2003), and $M_* \propto \sigma^{3.4 \pm 0.5}$, again similar to the Faber-Jackson relation estimated for global populations (e.g. Bernardi et al. 2003a). In any case, changing these slopes slightly (or e.g. using the $M_{\text{BH}}-\sigma$ relation of Ferrarese & Merritt (2000), as opposed to our best-fit which is very similar to Tremaine et al. (2002)) has a relatively small systematic effect on the best-fit residual slopes (this generally makes the trend in residuals stronger, because the ‘mean’ relation being subtracted is slightly less accurate for this particular sample), but does not qualitatively change our conclusions.

Of course, even this approach could in principle introduce a bias via our assumption of some functional form, and so we have also considered a non-parametric approach where we take the mean $\langle \log(M_{\text{BH}}) \rangle$ in bins of $\log(\sigma)$. Unfortunately, the small number of observations limits such a non-parametric technique and somewhat smears out the interesting correlations (slightly decreasing their statistical significance). Regardless, the lack of evidence for a non-linear (i.e. log-quadratic or higher-order) relation means that we find similar results in all these cases, so we conclude that our methodology in determining residuals is not introducing a significant bias.

The figure demonstrates that there is a highly significant correlation between M_{BH} and R_e or M_* at fixed σ . We repeat this exercise in the figure, and demonstrate similarly that there is a highly significant correlation between M_{BH} and σ or M_* at fixed R_e , and between M_{BH} and σ or R_e at fixed M_* . This indicates that a simple one-variable correlation (e.g. a $M_{\text{BH}}(\sigma)$, or $M_{\text{BH}}(M_*)$, $M_{\text{BH}}(R_e)$ relation) is an incomplete description of the observations. We therefore introduce a FP-like relation of the form

$$M_{\text{BH}} \propto \sigma^\alpha R_e^\beta, \quad (3)$$

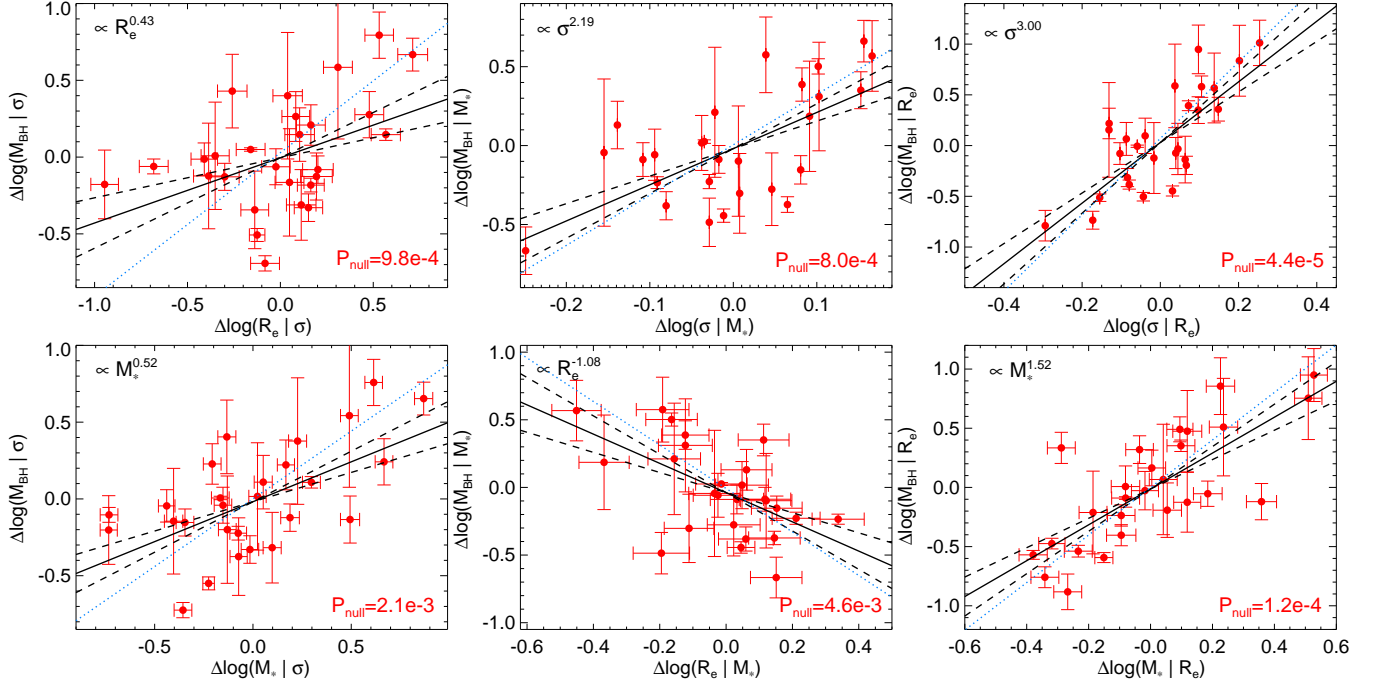


FIG. 2.— *Left*: Observed residual in BH mass as a function of host galaxy effective radius R_e (top) or stellar mass M_* (bottom), at fixed velocity dispersion σ (equivalently, correlation between the residuals in the $M_{\text{BH}} - \sigma$ and $R_e - \sigma$ or $M_* - \sigma$ relations at each σ). The fit to this residual correlation is shown with the black lines ($\pm 1\sigma$ range in the best-fit correlation shown as dashed lines – note that they are strongly inconsistent with zero correlation), with the slope shown (dotted blue line shows the least-squares bisector). The probability of the null hypothesis of no correlation in the residuals (i.e. no systematic dependence of M_{BH} on R_e or M_* at fixed σ) for the observed systems is shown (red P_{null}). *Center*: Residual in M_{BH} as a function of σ or R_e at fixed stellar mass M_* . *Right*: Residual in M_{BH} in M_{BH} as a function of σ or M_* at fixed effective radius R_e . The observations imply a secondary “fundamental plane”-type correlation at 3σ with respect to each of these variables.

which can account for these dependencies. Formally, we determine the combination of (α, β) which simultaneously minimizes the χ^2/ν of the fit and the significance of the correlations between the residuals in σ and M_{BH} (or R_e and M_{BH}). This yields similar results to the direct fitting method of Bernardi et al. (2003b) from the spheroid FP, which minimizes

$$\Delta^2 = [\log(M_{\text{BH}}) - \alpha \log(\sigma) - \beta \log(R_e) - \delta]^2. \quad (4)$$

It is straightforward to extend this minimization by weighting each point by the measurement errors (where we allow for the errors in all observed quantities – $\log(M_{\text{BH}})$, $\log(\sigma)$, and $\log(R_e)$, and estimate symmetric errors as the mean of quoted two-sided errors). This yields a best-fit BHFP relation

$$\log(M_{\text{BH}}) = 8.33 + 3.00(\pm 0.30) \log(\sigma/200 \text{ km s}^{-1}) + 0.43(\pm 0.19) \log(R_e/5 \text{ kpc}) \quad (5)$$

from the observations. Unsurprisingly, the slopes in the BHFP relation are close to those formally determined for the residuals in Figure 2. As expected, the residuals of M_{BH} with respect to these fundamental plane relations, at fixed R_e and fixed σ , show no systematic trends and are consistent with small intrinsic scatter. The introduction of a BHFP eliminates the strong systematic correlations between the residuals, yielding flat errors as a function of σ and R_e .

At low redshift, σ , R_e , and M_{dyn} can be determined reliably, but at high redshift it is typically the stellar mass M_* or luminosity which is used to estimate M_{BH} . Therefore, it is interesting to examine the BHFP projections in terms of e.g. M_* and σ or M_* and R_e . Repeating our analysis, we find in Figure 2 that the observations demand a FP relation over a simple $M_{\text{BH}}(M_*)$ relation at high significance. The exact values of the best-fit coefficients of this BHFP determined from

TABLE 1
BH-HOST CORRELATIONS

Variables ¹	Normalization ²	Slope ³	Scatter ⁴
$\sigma^\alpha R_e^\beta$	8.33 ± 0.06	$3.00 \pm 0.30, 0.43 \pm 0.19$	0.21
$M_*^\alpha \sigma^\beta$	8.24 ± 0.06	$0.54 \pm 0.17, 2.18 \pm 0.58$	0.22
$M_*^\alpha R_e^\beta$	8.06 ± 0.07	$1.78 \pm 0.40, -1.05 \pm 0.37$	0.25
$M_* \sigma^2$	8.23 ± 0.06	0.71 ± 0.06	0.25
σ	8.28 ± 0.08	3.96 ± 0.39	0.31
M_*	8.21 ± 0.07	0.98 ± 0.10	0.33
M_{dyn}	8.22 ± 0.10	1.05 ± 0.13	0.43
R_e	8.44 ± 0.10	1.33 ± 0.25	0.45

¹ For the variables (x, y) , a correlation of the form $\log(M_{\text{BH}}) = \alpha \log(x) + \beta \log(y) + \delta$ is assumed, where the normalization is δ and α, β are the logarithmic slopes.² The normalization gives $\log(M_{\text{BH}})$ for $\sigma = 200 \text{ km s}^{-1}$, $M_* = 10^{11} M_\odot$, $M_{\text{dyn}} = 10^{11} M_\odot$, $R_e = 5 \text{ kpc}$, which roughly minimizes the covariance between fit parameters.³ Errors quoted here for the BHFP relations in (σ, R_e) , (M_*, σ) , and (M_*, R_e) include the covariance between the two slopes. Holding one of the two fixed and varying the other yields substantially smaller errors (typically $\sim 5\%$). All quoted errors account for measurement errors in both M_{BH} and the relevant independent variables.⁴ The internal scatter is estimated from the observations as that which yields a reduced $\chi^2/\nu = 1$ with respect to the given best-fit relation.

the observations are given (along with those of various other BHFP projections) in Table 1.

The BHFP in terms of σ and R_e (i.e. $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$) is of course tightly related to the BHFP in terms of σ and M_* ($M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$) or M_* and R_e ; the near-IR fundamental plane relates stellar mass (assuming K -band luminosity is a good proxy for stellar mass), effective radius, and velocity dispersion as $R_e \propto \sigma^{1.53} I_e^{-0.79}$ (Pahre et al. 1998). Using $I_e \propto M_*/R_e^2$, we can substitute this in Equation (5) and obtain the expected BHFP in terms of σ and M_* , namely $M_{\text{BH}} \propto \sigma^{1.9} M_*^{0.58}$. This is quite close to the result of our direct fitting, $M_{\text{BH}} \propto \sigma^{2.2} M_*^{0.54}$. The FP of early-type galaxies, in other words, allows us to

relate our two “fundamental planes,” namely $M_{\text{BH}} \propto \sigma^3 R_e^{1/2}$ and $M_{\text{BH}} \propto \sigma^2 M_*^{1/2}$. Given the tight early-type FP relation between M_* , σ , and R_e , these two forms of the BHFP are completely equivalent (the choice between them is purely a matter of convenience). We can, in fact, pick any two of the three FP-related variables as our independent variables for predicting M_{BH} – using the early-type FP again to transform the BHFP relation in terms of σ and R_e to one in terms of M_* and R_e , we expect $M_{\text{BH}} \propto M_*^{1.6} R_e^{-0.8}$, similar to the relation we directly fit (see Table 1). Any two of M_* , R_e , and σ can thus be used to predict M_{BH} according to the BHFP relations. The tightness of the early-type FP also means that it is redundant to search for a four-variable correlation (i.e. one of the form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta M_*^\gamma$), since M_* is itself a function of σ and R_e with very little scatter (and indeed, directly testing this, we find no significant improvement in our fits expanding to a four-variable correlation). Perhaps most important, however, is that no transformation (given the early-type FP relating these three variables) eliminates the dependence on two variables – i.e. no transformation from any one BHFP allows us to write M_{BH} as a pure function of either σ , R_e , M_* , or M_{dyn} (as we expect, since Figure 2 explicitly shows that each of these single-variable correlations exhibits a significant residual dependence on another variable).

Given the definition of $M_{\text{dyn}} \propto \sigma^2 R_e$, it is trivial to convert the best-fit BHFP relation in terms of σ and R_e to one in M_{dyn} , obtaining

$$\begin{aligned} \log(M_{\text{BH}}) &= 8.29 + 0.43 \log(M_{\text{dyn}}/10^{11} M_\odot) \\ &\quad + 2.14 \log(\sigma/200 \text{ km s}^{-1}) \\ &= 8.19 + 1.50 \log(M_{\text{dyn}}/10^{11} M_\odot) \\ &\quad - 1.07 \log(R_e/5 \text{ kpc}). \end{aligned} \quad (6)$$

It is important to note that the residual correlation with either σ or R_e at fixed M_{dyn} is non-zero and highly significant. It is therefore not the case that the BHFP reflects, for example, the true correlation being between M_{BH} and M_{dyn} (in which case the BHFP would have a form $M_{\text{BH}} \propto (\sigma^2 R_e)^\gamma$), but it is a FP in a genuine sense.

We have also repeated the analysis of Figure 2 for M_{dyn} and σ (or M_{dyn} and R_e), and obtain these results directly, with a pure correlation between M_{BH} and M_{dyn} ruled out at $\sim 3\sigma$ in the observations. We note that any analysis of these particular residual correlations must be done with special care, as the plotted quantities (e.g. $M_{\text{BH}}/M_{\text{dyn}}$ versus $R_e/(R_e[M_{\text{dyn}}])$) are *not* independent (since $M_{\text{dyn}} \propto \sigma^2 R_e/G$ depends directly on the measured R_e), and so fitting a small sample where there is both intrinsic scatter and measurement errors in the quantities can bias the fit. Running a series of Monte Carlo experiments allowing for the range of estimated intrinsic scatter and measurement errors in each quantity, we find that the observations span a sufficiently small baseline that, with the present errors, a naive comparison will be biased to *underestimate* the significance of the preference for a BHFP relation over a simple correlation with M_{dyn} . The P_{null} for correlations between the residuals in the $M_{\text{BH}} - M_{\text{dyn}}$ relation and R_e or σ typically decreases by a factor ~ 2 with a proper Monte Carlo analysis; i.e. the true significance of the BHFP relation is even greater than a direct comparison suggests.

Figure 2 demonstrates the significance with which the observations rule out *both* a pure BH-host mass relation (either $M_{\text{BH}} \propto M_*$ or $M_{\text{BH}} \propto M_{\text{dyn}}$) and a pure $M_{\text{BH}} - \sigma$ relation. However, when fitting to a form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$, there is still some

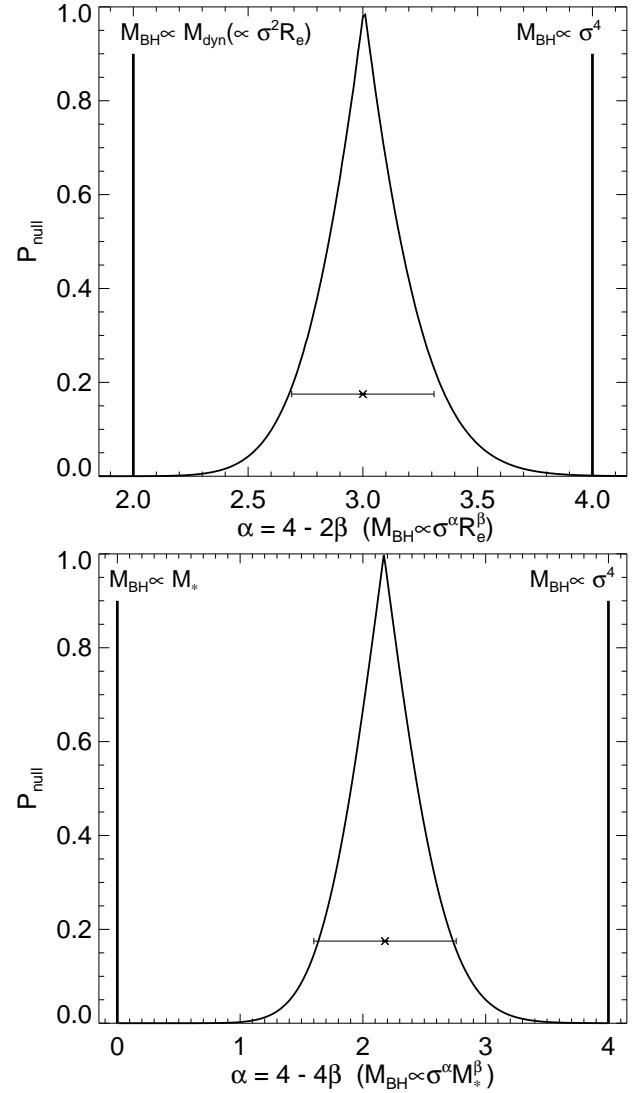


FIG. 3.— Probability that there is no remaining trend or correlation among the residuals of the M_{BH} -host relation (P_{null} , as in Figure 2), as a function of the slope α for a correlation of the form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$ (top) or $M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$ (bottom). For each value of α , we marginalize over β and the correlation normalization to find the best fit, but there is a rough degeneracy between the best fit α and β ($\beta \approx (4-\alpha)/2$ for $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$, $\beta \approx (4-\alpha)/4$ for $M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$). Lines show P_{null} for the observations, points show the best fit (in a χ^2 sense) BHFP relation and errors from Table 1. Thick black lines show the α corresponding to a pure $M_{\text{BH}} - \sigma$, $M_{\text{BH}} - M_{\text{dyn}}$, or $M_{\text{BH}} - M_*$ relation, all of which are ruled out at similar ($\sim 3\sigma$) significance.

degeneracy between the slopes α and β (roughly along the axis $\beta \approx (4-\alpha)/2$). Figure 3 illustrates the degree of this degeneracy and the extent to which, for example, a BHFP with $M_{\text{BH}} \propto \sigma^3 R_e^{1/2}$ is favored over a pure $M_{\text{BH}} - M_{\text{dyn}}$ relation. We plot the likelihood of a residual correlation between M_{BH} and R_e or σ at fixed $\sigma^\alpha R_e^\beta$, as a function of the slope α (marginalizing over β and other fit parameters at each α , although $\beta(\alpha)$ roughly follows the axis of degeneracy above). In detail, this is the likelihood of a correlation between the residuals of the $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$ relation and the residuals of the $\sigma(\sigma^\alpha R_e^\beta)$ and $R_e(\sigma^\alpha R_e^\beta)$ relations, identical to the procedure used to determine the quoted P_{null} in Figure 2. We compare the quoted best-fit slopes from Table 1. We then repeat this exercise for the alternate representation of the fundamental plane, $M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$.

The analysis shown in Figure 3 agrees reasonably well with the χ^2 best-fit expectations, and illustrate an important point: a pure relation between BH and host mass (either dynamical mass M_{dyn} or stellar mass M_*) is ruled out at a significance level comparable to that with which a pure $M_{\text{BH}} - \sigma$ relation is ruled out ($\sim 3\sigma$). Indeed, the preferred BHFP parameters are centered almost exactly mid-way between these two previously proposed relations.

However, there are possible correlations which cannot be clearly discriminated by the present data. Both a pure relation between BH mass and spheroid binding energy, of the form $M_{\text{BH}} \propto (M_* \sigma^2)^{2/3}$, as studied in Aller & Richstone (2007), for example, and a mixed relation of the form $M_{\text{BH}} \propto M_*^{1/2} \sigma^2$ (see Hopkins et al. 2007), are within the $\sim 1\sigma$ allowed range of the data. It is worth noting that simply expanding the number of observed sources will not necessarily break these degeneracies. Rather, to increase the constraining power of the observations, a larger baseline is needed, including (in particular) a larger sample of objects which lie off the mean $R_e - \sigma$ or $M_* - \sigma$ relations (and thus extend the baseline in the residual-residual space which properly constrains the FP slopes).

Finally, we noted above that Figure 2 is essentially unchanged if we consider residuals with respect to just linear (i.e. pure power law) $M_{\text{BH}} - \sigma$ -type relations, as a consequence of there being no significant evidence in our data for a log-quadratic or higher-order correlation. Allowing log-quadratic terms in our fundamental plane fits, we find a best fit to the observations of the form

$$\log(M_{\text{BH}}) = 8.06 + (2.8 \pm 0.4) \tilde{\sigma} + (0.48 \pm 0.18) \tilde{r} - (2.1 \pm 2.3) \tilde{\sigma}^2 + (0.31 \pm 0.25) \tilde{r}^2 \quad (7)$$

(where $\tilde{\sigma} \equiv \log[\sigma/200 \text{ km s}^{-1}]$ and $\tilde{r} \equiv \log[R_e/3 \text{ kpc}]$). The linear BHFP coefficients in σ and R_e are similar to those in Table 1, and their significance is not much changed – this illustrates that the FP behavior we find cannot simply trade off with or be equally well-represented by a log-quadratic dependence (i.e. one cannot eliminate the residual dependence of M_{BH} on R_e at fixed σ by adding a log-quadratic or higher-order term in σ). The log-quadratic terms are at most significant at the $\sim 1\sigma$ level. This is also true if we add just one of the two log-quadratic terms – adding a log-quadratic term in just R_e yields a coefficient $(0.16 \pm 0.25) \tilde{r}^2$, and adding one in just σ gives a coefficient $(-0.28 \pm 0.15) \tilde{\sigma}^2$. This is similar to the finding of Wyithe (2006), who estimates $< 1\sigma$ significance for the addition of log-quadratic terms in any of σ , M_{dyn} , or M_* .

4. CONCLUSIONS

We study the correlation between observed central BH mass and host galaxy properties, and find that the systems lie on a BH “fundamental plane” (BHFP), of the form $M_{\text{BH}} \propto \sigma^{3.0} R_e^{0.5}$ or $M_{\text{BH}} \propto M_*^{0.5-0.7} \sigma^{1.5-2.0}$, analogous to the FP of spheroids. Specifically, there are significant (at $> 99.9\%$ confidence) trends in the residuals of the $M_{\text{BH}} - \sigma$ relation with M_* and R_e at fixed σ , and likewise in the $M_{\text{BH}} - M_*$ relation (with σ or R_e at fixed M_*). This provides a new paradigm for understanding the traditional relations between BH mass and either

bulge velocity dispersion or mass. These correlations (as well as those with other bulge properties such as effective radius, central potential, dynamical mass, concentration, Sersic index, and bulge binding energy) are all projections of the same fundamental plane relation. Just as the Faber-Jackson relation between e.g. stellar mass or luminosity and velocity dispersion ($M_* - \sigma$) is understood as a projection of the more fundamental relation between M_* , σ , and R_e , so too is the $M_{\text{BH}} - \sigma$ relation ($M_{\text{BH}} \propto \sigma^4$) a projection of the more fundamental relation $M_{\text{BH}} \propto \sigma^3 R_e^{0.5}$. Recognizing this resolves the nature of several apparent outliers in the $M_{\text{BH}} - \sigma$ relation, which simply have unusual σ values for their stellar masses or effective radii, and eliminates the strong correlations between residuals.

Improved measurements of the host properties of systems with well-measured BHs can significantly improve constraints on the BHFP. As noted in Table 1, the present observations demand a correlation of the form $M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$ over a simple correlation with either σ or M_* at $\gtrsim 3\sigma$ confidence. Already, this puts strong constraints on theoretical models of BH growth and evolution – BH mass does not simply scale with the star formation (stellar mass) or virial velocity of the host galaxy. However, there is still a substantial degeneracy between the slopes α and β (roughly along the axis $\beta \approx 1 - \alpha/4$). For example, the existing data do not allow us to significantly distinguish a pure correlation with spheroid binding energy $M_{\text{BH}} \propto (M_* \sigma^2)^{2/3}$, as detailed in Aller & Richstone (2007) from the marginally favored relation $\propto M_*^{1/2} \sigma^2$. Both suggest that the ability of BHs to self-regulate their growth must be sensitive to the potential well at the center of the galaxy (and therefore to galactic structure), but the difference could reveal variations in the means by which BH feedback couples to the gas on these scales.

Increasing the observed sample sizes and, in particular, extending the observed baselines in mass and σ will substantially improve the lever arm on these correlations. In particular, the addition of stellar mass M_* information to the significant number of objects which have measurements of σ and indirect measurements of M_{BH} from reverberation mapping would enable considerably stronger tests of our proposed BHFP relation. We do note the caveat, however, that care should still be taken to consider only bulge properties and remove e.g. rotationally supported contributions to the velocity dispersion.

The BHFP appears to be a robust correlation, which provides an improved context in which to understand the nature and evolution of the numerous observed correlations between BH and host spheroid properties. In particular, the results described here provide new, important constraints for models of BH growth, feedback and self-regulation.

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APPENDIX

DETERMINING THE SIGNIFICANCE OF RESIDUALS IN THE M_{BH} -HOST CORRELATIONS

We test the robustness of our results, and how appropriate our fitting method is, by constructing a series of Monte Carlo realizations of the observations. As the velocity distribution σ is the most well-measured quantity for the observed galaxies (with

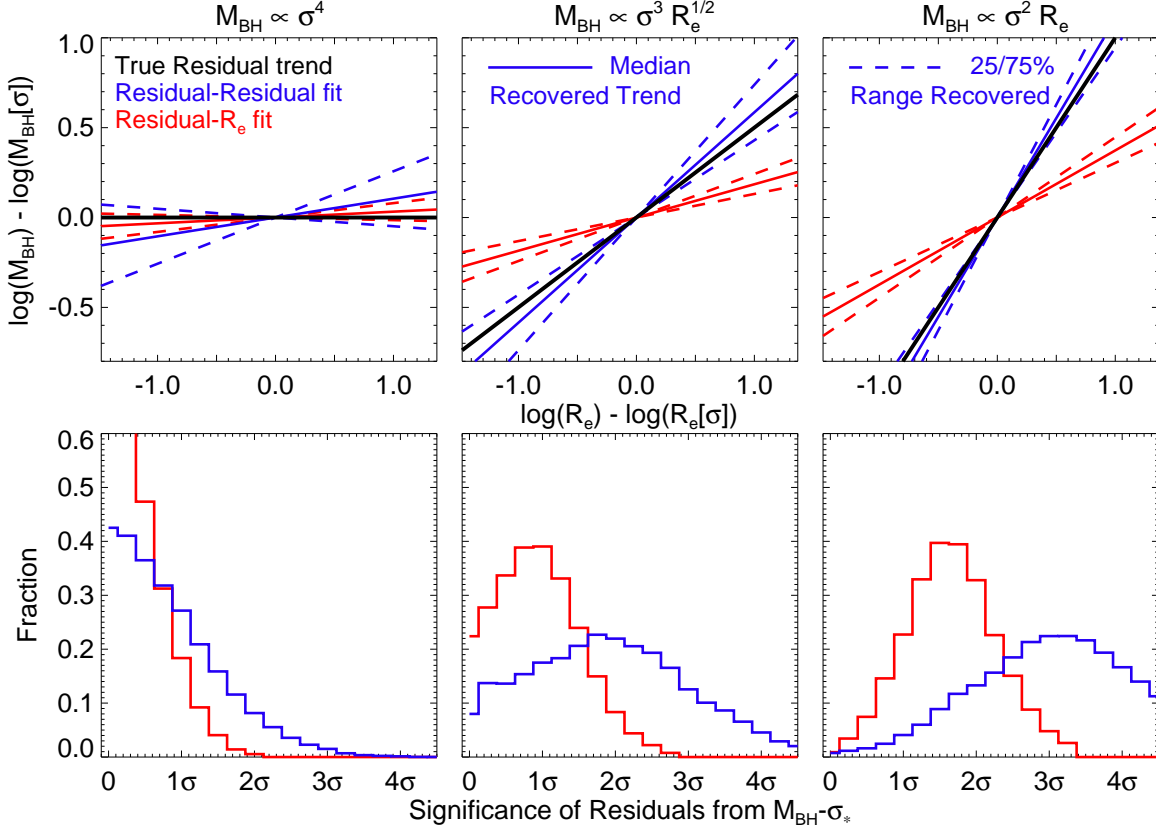


FIG. 4.— *Top*: Recovered dependence of M_{BH} on R_e at fixed σ (blue lines) from Monte Carlo realizations of the observed data sets, assuming different “true” intrinsic correlations between M_{BH} and host properties (of the form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$). Black lines in each panel show the residual of the assumed “true” relation, i.e. zero dependence in the case where the true relation is $M_{\text{BH}} \propto \sigma^4$ (left), residual $M_{\text{BH}} \propto R_e^{1/2}$ for a BHFP relation of the form $M_{\text{BH}} \propto \sigma^3 R_e^{1/2}$ (center), and residual $M_{\text{BH}} \propto R_e$ for a $M_{\text{BH}} \propto M_{\text{dyn}}$ relation (right). Solid blue lines show the median dependence recovered using our standard fitting method (fitting residuals at fixed σ) for all Monte Carlo realizations, dashed lines show the 25/75% range of fits recovered. If we fit the residuals as a function of the actual value of R_e (instead of R_e at fixed σ), we infer the correlations shown as red lines. In all cases, fitting residuals versus residuals (our standard methodology) recovers a similar slope to the “true” correlation and appropriate (in a χ^2 sense for normal errors) range about that true slope. However, comparing to the actual value of R_e severely smears out the true dependence on R_e when it is present. *Bottom*: The distribution of the significance of the residuals (of M_{BH} with respect to R_e) in our Monte Carlo realizations. The same trends are seen: fitting residuals versus residuals yields the expected χ^2 distributions and generally detects the appropriate significance of residuals when there is a true dependence on R_e , but in almost all cases (even when the true correlation has a strong dependence on R_e), fitting residuals in M_{BH} against the actual value of R_e leads one to conclude (incorrectly) that the residuals are not significant.

the exception of the Milky Way, which we generally exclude from our fits following Tremaine et al. (2002)), we begin with the observed distribution of σ values. Either from the cumulative distribution of σ values, or from the observed error bars for each value of σ , we then statistically resample the σ distribution (with the same number of objects) for each Monte Carlo realization. Next, we assume a mean intrinsic correlation between R_e and σ (for simplicity, assume $R_e \propto \sigma^2$, similar to that observed), with some intrinsic scatter comparable to that observationally inferred, and use this to randomly generate the true R_e values of each point. Then, given some assumed true BH-host correlation of the form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$, again with an intrinsic scatter comparable to the observational estimates, we randomly generate the true BH masses. Finally, using the mean observational measurement errors in each of these quantities (or, specifically the quoted observational errors of each object corresponding to each mock point, as it makes little difference), we randomly generate the “observed” values of each quantity. We then repeat our fitting procedures from § 3. For example, to search for residuals with respect to the $M_{\text{BH}} - \sigma$ relation, we fit the “observed” points to a mean $M_{\text{BH}} - \sigma$ relation, and compare the residuals in BH mass to the residuals in R_e of the “observed” $R_e - \sigma$ relation. We determine the statistical significance of these residuals, and fit to determine the residual dependence (i.e. best fit “observed” $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$ relation).

Figure 4 shows the results of this analysis. Specifically, we show the median correlations between the residuals in M_{BH} and in R_e , from ~ 1000 Monte Carlo realizations, and the 25/75% quartile ranges of the fitted correlations. We show this for three assumed cases – one in which the “true” correlation is a pure $M_{\text{BH}} - \sigma$ relation ($M_{\text{BH}} \propto \sigma^4$), one in which the true correlation is similar to our best-fit BHFP relation ($M_{\text{BH}} \propto \sigma^3 R_e^{1/2}$), and one in which the true correlation is a pure $M_{\text{BH}} - M_{\text{dyn}}$ relation ($M_{\text{BH}} \propto M_{\text{dyn}} \propto \sigma^2 R_e$). In all of these cases, our fitting method generally recovers a residual correlation very similar to the input correlation. There appears to be a (very slight) bias towards our fitting method recovering a marginally stronger dependence of M_{BH} on R_e , however, in terms of its statistical significance, the bias introduced is very small – at most, it implies that the “true” (in a maximum likelihood sense) significance of the dependence of M_{BH} on R_e at fixed σ which we recover from the observations should be slightly lower than quoted (equivalent to increasing our error bars by $\sim 10\%$). In other words, the observational data span a sufficient baseline, and the fitting method we use is sufficiently robust, that any “true” correlation of the form $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$ should be recovered in the majority of cases.

We can also check the significance of the residuals (of M_{BH} with respect to R_e) from each of these Monte Carlo realizations, as shown in Figure 4. In the case of a pure “true” $M_{\text{BH}} - \sigma$ relation, the residuals should not be significant in most cases, and indeed we find that they are not. Their behavior is essentially exactly what is expected for a χ^2 distribution – i.e. in 5% of all cases, a 95% significance level is assigned to the residuals, in 1% of all cases, a 99% significance level is assigned. The significance as we have determined it therefore carries its appropriate (expected) meaning and weight, and our general χ^2 analysis and assumption of normal errors in log-space is probably not misleading. In the case where the “true” correlation is of the form $M_{\text{BH}} \propto \sigma^3 R_e^{1/2}$ or $M_{\text{BH}} \propto \sigma^2 R_e$, the likelihood that the residuals will be significant increases dramatically (as it should), although we caution that there is still some (fairly large) probability that the residuals of a given Monte Carlo realization will not be especially significant ($\lesssim 2\sigma$ significance). From comparison with the residuals with respect to a pure $M_{\text{BH}} - \sigma$ relation, this implies that the parameter space spanned by current observations is such that a significant detection of residuals *does* imply a significant probability of a true BHFP relation, but that a weak or non-detection of such residuals *does not yet rule out* such a relation.

We caution, however, that this is only the case when comparing residuals against residuals. If we were, as in Figure 1, to simply consider the residual in $M_{\text{BH}} - \sigma$ as a function of the actual value of R_e (instead of comparing M_{BH} and R_e both at fixed σ), we smear out the significance of any real residuals, as we would expect. The slope recovered (i.e. the inferred dependence of M_{BH} on R_e) is severely biased towards being too shallow for any non-zero dependence on R_e , and in only $\sim 1\%$ of cases will such a method recover a slope similar to the “true” intrinsic correlation. Looking at the significance of the residuals in this space, it is clear that this projection biases against detecting any significant residual dependence on R_e . Even if the true relation were $M_{\text{BH}} \propto \sigma^2 R_e$, looking at the significance of the residuals as a function of the actual value of R_e would lead one to conclude in over 80% of cases that they were insignificant ($< 2\sigma$ formal significance). A proper analysis, however, would recover the true significance of the residuals in $\gtrsim 95 - 99\%$ of cases.

Finally, by repeating this analysis over the entire parameter space of “true” correlations (for example forms $M_{\text{BH}} \propto \sigma^\alpha R_e^\beta$), and comparing with the data, we can use a similar Monte Carlo approach to determine a maximum-likelihood, best-fit BHFP relation. We have done so, and find nearly identical results to our standard fits, again indicating that the conclusions herein are robust to the exact likelihood calculation so long as the residuals are properly analyzed. Similarly, we can repeat this entire analysis for true correlations of the form $M_{\text{BH}} \propto \sigma^\alpha M_*^\beta$ or $M_{\text{BH}} \propto M_*^\alpha R_e^\beta$, or in order to test the significance of residuals at fixed R_e or fixed M_* . In all cases, we recover very similar results, reinforcing the robustness of our conclusions.

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